

Nonrelativistic effective field theory of unstable top

Alexander A. Penin^{*ab}, Jan H. Piclum^{cd}

^a*Department of Physics, University of Alberta,
Edmonton AB T6G 2J1, Canada*

^b*Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT),
76128 Karlsruhe, Germany*

^c*Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen,
52056 Aachen, Germany*

^d*Physik Department T31, Technische Universität München,
85748 Garching, Germany*

E-mail: penin@ualberta.ca, jan.piclum@tum.de

We develop a new nonrelativistic effective field theory of ρ NRQCD [1] to describe the finite lifetime effects in the threshold production of top quark-antiquark pairs. The theory is based on the expansion in a parameter $\rho = 1 - m_W/m_t$ characterizing the dynamics of the top-quark decay. Within this framework we compute the nonresonant contribution to the total cross section of the top quark-antiquark threshold production in electron-positron annihilation up to the next-to-next-to-leading order. Our method naturally resolves the problem of spurious divergences in the analysis of the unstable top production.

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^{*}Speaker.

1. Introduction

The threshold production of top quark-antiquark pairs at a future linear collider may provide us with the most accurate information on the top-quark mass and couplings crucial for our understanding of electroweak symmetry breaking and mass generation mechanism [2]. The non-relativistic top-antitop pair is the cleanest quarkonium-like system and its theoretical description is entirely based on the first principles of QCD [3–5]. Over the last decade a significant progress has been achieved in the analysis of the higher order perturbative and relativistic corrections in the strong coupling constant α_s and the heavy-quark velocity v to the threshold cross section. Sizeable next-to-next-to-leading order (NNLO) corrections have been found by several groups [6–11] that stimulated the study of the higher orders of perturbation theory. Currently a large part of the third order corrections is available [12–26] and the N³LO analysis is likely to be completed in the foreseeable future. Moreover the higher order logarithmically enhanced corrections have been resummed through the effective theory renormalization group methods [27–29].

At the same time much less attention has been paid to the analysis of the effects related to the instability of the top quark [30–33]. The width of the electroweak top-quark decay $t \rightarrow W^+b$, $\Gamma_t \approx 1.5$ GeV, is comparable to the binding energy of a would-be toponium ground state and has a dramatic effect on the threshold production. It serves as an infrared cutoff, which makes the process perturbative in the whole threshold region, and smears out all the Coulomb-like resonances below the threshold leaving a single well pronounced peak in the cross section. The standard prescription in the analysis of the unstable top-quark production consists of the complex shift $E \rightarrow E + i\Gamma_t$, where E is the top-antitop pair energy counted from the threshold [3]. Though this procedure incorporates the dominant effect of the finite top-quark width, it does not fully account for nonresonant processes like $e^+e^- \rightarrow tW^-\bar{b}$, $e^+e^- \rightarrow bW^+\bar{t}$ or $e^+e^- \rightarrow W^+W^-b\bar{b}$ where the intermediate top quark is not on its (complex) mass shell. Such processes *cannot be distinguished* from the resonant $t\bar{t}$ production, which has the same final states due to the top-quark instability.

Recently different approaches have been suggested to refine the analysis of the finite width effect [1, 34, 35]. The method of ref. [1] is based on a new nonrelativistic effective theory which we name ρ NRQCD. In this proceeding we outline its main idea and present the results obtained within this framework.

2. Finite width effect beyond the complex energy shift

In the Born approximation the total cross section of top-antitop production in electron-positron annihilation is related through the optical theorem to the imaginary part of the one-loop forward scattering amplitude. The corresponding expression for the normalized cross section $R = \sigma(e^+e^- \rightarrow t\bar{t})/\sigma_0$, $\sigma_0 = 4\pi\alpha^2/(3s)$, in the threshold region $s \approx 4m_t^2$ can be obtained by the standard nonrelativistic expansion of the top-quark vertices and propagators in v and reads

$$R_{res}^{Born} = \left[Q_e^2 Q_t^2 + \frac{2Q_e Q_t v_e v_t}{1 - x_Z} + \frac{(a_e^2 + v_e^2) v_t^2}{(1 - x_Z)^2} \right] \frac{6\pi N_c}{m_t^2} \text{Im}[G_0(0, 0, E + i\epsilon)] + \dots, \quad (2.1)$$

where the ellipsis stands for the relativistic corrections, Q_f is the electric charge of fermion f in units of the positron charge, $N_c = 3$ is the number of colors, and $x_Z = m_Z^2/(4m_t^2)$ with the Z -boson

mass m_Z . The couplings of fermion f to the Z -boson are

$$v_f = \frac{I_{w,f}^3 - 2s_w^2 Q_f}{2s_w c_w}, \quad a_f = \frac{I_{w,f}^3}{2s_w c_w}, \quad (2.2)$$

where $I_{w,f}^3$ is the third component of the fermion's weak isospin and s_w (c_w) is the sine (cosine) of the weak mixing angle. Note that only the vector coupling of the top quark gives the leading order contribution and the axial coupling is suppressed by an additional power of v . The last factor in eq. (2.1) is

$$G_0(0,0,E) = \int \frac{d^{d-1}\vec{p}}{(2\pi)^{d-1}} \frac{m_t}{\vec{p}^2 - m_t E} = -\frac{m_t^2}{4\pi} \sqrt{\frac{-E}{m_t}}, \quad (2.3)$$

which is nothing but the Green's function of the free Schrödinger equation at the origin. Formally the integral in eq. (2.3) is linearly divergent but the divergent part is real and does not contribute to the cross section. To handle the divergence we use dimensional regularization with $d = 4 - 2\varepsilon$, where the integral (2.3) is finite even for $\varepsilon = 0$. The strong interaction has a significant impact on the threshold cross section. Close to threshold when $v \sim \alpha_s$ the Coulomb effects become nonperturbative and have to be resummed to all orders in α_s by substituting eq. (2.3) with the full Coulomb Green's function

$$G_C(0,0;E) = G_0(0,0;E) + G_1(0,0;E) - \frac{C_F \alpha_s m_t^2}{4\pi} \left[\Psi \left(1 - \frac{C_F \alpha_s}{2} \sqrt{\frac{m_t}{-E}} \right) + \gamma_E \right], \quad (2.4)$$

where Ψ is the logarithmic derivative of the Gamma function and $C_F = 4/3$. The one-gluon exchange contribution $G_1(0,0;E)$ is ultraviolet divergent. Again for stable top quarks the divergent part is real and does not contribute to eq. (2.1). In the $\overline{\text{MS}}$ subtraction scheme this term reads

$$G_1(0,0;E) = -\frac{C_F \alpha_s m_t^2}{8\pi} \left[\ln \left(\frac{-m_t E}{\mu^2} \right) - 1 + 2 \ln 2 \right]. \quad (2.5)$$

Let us now consider the top-quark decay. Every decay process is suppressed by the electroweak coupling constant α_{ew} . We adopt the standard power counting rules $\alpha_s \sim v$, $\alpha_{ew} \sim v^2$. Thus to NNLO if the top quark decays the antiquark may be treated as a stable particle and vice versa. The dominant effect of the top-quark instability is related to the imaginary part of its mass operator in diagram 1(a). In the massless bottom quark approximation and with the off-shell momentum p the mass operator reads

$$\text{Im}[\Sigma^{(0)}(p^2)] = \frac{G_F}{16\pi\sqrt{2}} p^3 \left(1 + 2 \frac{m_W^2}{p^2} \right) \left(1 - \frac{m_W^2}{p^2} \right)^2 \theta(p^2 - m_W^2), \quad (2.6)$$

where G_F is the Fermi constant and we use the approximation $V_{tb} = 1$. Close to the mass shell one has $p^2 = m_t^2 - 2(\vec{p}^2 - m_t E) + \dots$, where \vec{p} is the spatial momentum of the top quark, and the mass operator can be expanded in $z = (\vec{p}^2 - m_t E)/m_t^2 \ll 1$

$$\begin{aligned} \text{Im}[\Sigma^{(0)}(z)] &= \frac{\Gamma_t}{2} \left(1 - \frac{4z}{(1-x^2)} + \frac{4z^2}{(1-x^2)^2} \right) \theta(1-x^2-2z) + \dots \\ &= \frac{\Gamma_t}{2} - \frac{\Gamma_t}{2} \left[\theta(x^2+2z-1) + \left(\frac{4z}{(1-x^2)} - \frac{4z^2}{(1-x^2)^2} \right) \theta(1-x^2-2z) \right] + \dots \end{aligned} \quad (2.7)$$

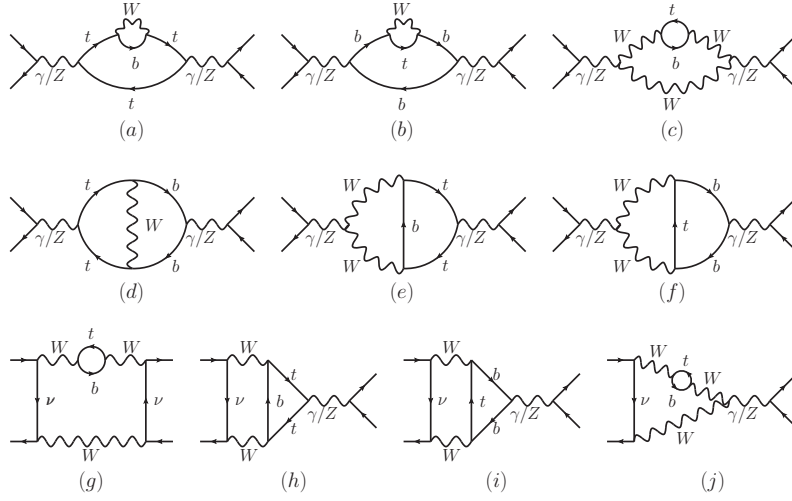


Figure 1: e^+e^- forward scattering diagrams containing $bW^+\bar{t}$ and $tW^-\bar{b}$ cuts.

where $x = m_W/m_t$, $\Gamma_t = \Gamma_t^{(0)} + \mathcal{O}(\alpha_s)$ and

$$\Gamma_t^{(0)} = \frac{G_F m_t^3}{8\pi\sqrt{2}} (1 + 2x^2)(1 - x^2)^2, \quad (2.8)$$

is the leading order top-quark electroweak width. Note that in the expansion (2.7) we consider $1 - x = \rho$ to be of the same order of magnitude as z . The first term in the last line of eq. (2.7) describes the standard shift of the pole position of the top quark propagator into the unphysical sheet of the complex energy plane characteristic for unstable particles. After Dyson resummation it replaces the argument of eq. (2.3) by $E + i\Gamma_t$, which is the original prescription of ref. [3]. Eq. (2.7), however, contains the remainder which also contributes to the imaginary part of the forward scattering amplitude. Since the remainder vanishes for on-shell top quark, it represents the nonresonant process $e^+e^- \rightarrow bW^+\bar{t}$ or $e^+e^- \rightarrow tW^-\bar{b}$. Direct evaluation of the nonresonant contribution is technically complex as the problem involves a large number of scales characterizing both the top-quark threshold dynamics and the dynamics of the top-quark decay. Such a problem, however, is an ideal application of the effective field theory approach.

3. Nonrelativistic effective theory of unstable top

In the nonresonant contribution the integral over the virtual momentum \vec{p} is saturated by the region $|\vec{p}| \sim \rho^{1/2}m_t$ where the argument of the θ -function in eq. (2.7) vanishes. The main idea of our approach is that if ρ is considered as a small parameter this momentum region corresponds to a nonrelativistic top quark with the energy $p_0 - m_t \sim \vec{p}^2/m_t \sim \rho m_t$. It is equivalent to the top quark momentum scaling in the standard potential nonrelativistic QCD (pNRQCD) [36, 37] with the heavy quark velocity v replaced by $\rho^{1/2}$. Thus beside the hard scale m_t , the soft scale ρm_t , and the ultrasoft scale $\rho^{1/2}m_t$ characterizing the top-quark threshold dynamics we have ρ -soft scale $\rho^{1/2}m_t$ and ρ -ultrasoft scale ρm_t characterizing the dynamics of the top-quark decay. In full

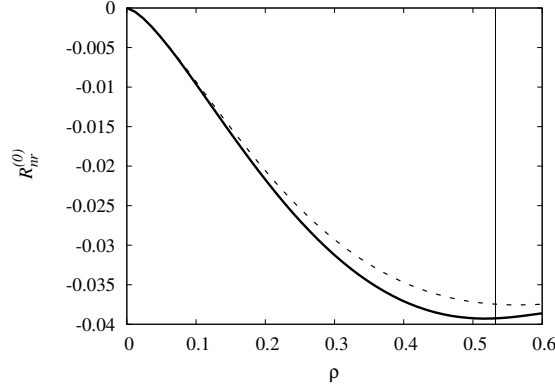


Figure 2: The NLO nonresonant contribution to the cross section(solid curve) and the leading order approximation(dashed line). The vertical line marks the physical value of ρ .

analogy with the pNRQCD the new scales give rise to the ρ -soft and ρ -ultrasoft modes as well as to the ρ -potential modes with the nonuniform scaling of energy and three-momentum $p_0 - m_t \sim \rho m_t$, $|\vec{p}| \sim \rho^{1/2} m_t$. The ρ NRQCD is a new nonrelativistic effective theory which incorporates these modes. To disentangle the ordinary pNRQCD and the new ρ NRQCD modes we impose the scale hierarchy $v m_t \ll \rho^{1/2} m_t \ll m_t$ or $v \ll \rho^{1/2} \ll 1$. The cross section is then constructed as a series in the scale ratios.

Eq. (2.7) does not give the full nonresonant contribution and one has to take into account all diagrams of this order with the $bW^+\bar{t}$ or $tW^-\bar{b}$ cuts given in figure 1. The calculation is significantly simplified within the nonrelativistic effective theory where all the propagators which are off-shell by the amount m_t collapse, giving rise to new effective theory vertices. From the practical point of view, however, it is more convenient to directly expand the full theory Feynman integrals in ρ through the expansion by regions [38, 39] than to use the effective theory Feynman rules. The contribution comes from a single region where the top quark and W -boson are ρ -potential and the bottom quark is ρ -ultrasoft and one may neglect all the terms suppressed by the ratio v^2/ρ , *i.e.* put $E = 0$. We found that beside diagram 1(a) only diagram 1(g) gives a leading order contribution in ρ . For the total nonresonant contribution to the cross section at the leading order in ρ we obtain a simple analytical result

$$R_{nr} = -\frac{8N_c}{\pi\rho^{1/2}} \frac{\Gamma_t}{m_t} \left[\left(Q_e^2 Q_t^2 + \frac{2Q_e Q_t v_e v_t}{1-x_Z} + \frac{(a_e^2 + v_e^2)v_t^2}{(1-x_Z)^2} \right) - \frac{1}{s_w^4} \left(\frac{17}{48} - \frac{9\sqrt{2}}{32} \ln(1+\sqrt{2}) \right) + \mathcal{O}(\rho, \alpha_s) \right]. \quad (3.1)$$

It is suppressed with respect to the leading resonant contribution (2.1) and according to the power counting rules represents a NLO contribution to the total cross section.

3.1 Relativistic and perturbative corrections

Eq. (3.1) gives the nonresonant contribution in the leading order of the nonrelativistic expansion in ρ and perturbative expansion in α_s . In the leading order in α_s it is straightforward to

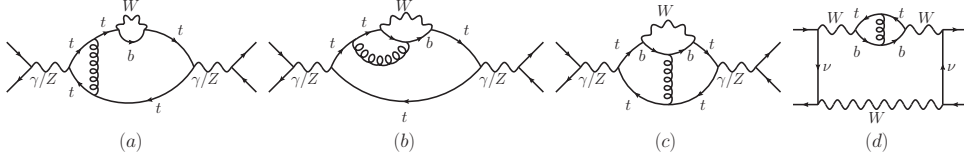


Figure 3: e^+e^- forward scattering diagrams contributing to the $\mathcal{O}(\alpha_s)$ correction in the leading order in ρ . The curly lines denote gluons.

compute a sufficient number of terms of the expansion in ρ of the diagrams in Fig. 1 to ensure very good accuracy of the approximation for the physical value $\rho \approx 0.53$ [1]. The Padé-improved series converges to the result of [35] obtained without the expansion. The result for the NLO contribution after summation of the series is plotted in figure 2 as function of ρ along with the leading order term (3.1). The latter turns out to be a good approximation in the whole interval $0 < \rho < 0.6$ and deviates from the total result by less than 5% at the physical value of ρ .

In the leading order in ρ the α_s corrections are obtained by the gluon dressing of diagrams 1(a) and 1(g) shown in figure 3. The ρ NRQCD scaling and power counting rules identify the relevant regions of virtual momentum. In particular, the diagram 3(a) gets contributions from the hard, potential, and ρ -potential regions of gluon momentum, the diagrams 3(b) and 3(d) get contributions from the hard and ρ -soft regions, while the diagram 3(c) vanishes in the leading order of the nonrelativistic expansion [31]. By adding up the contributions of all the regions one gets the $\mathcal{O}(\alpha_s)$ correction to eq. (3.1). It represents the NNLO nonresonant contribution to the total cross section in the leading order in ρ and can be obtained in a closed analytical form [1]

$$\begin{aligned}
 R_{nr}^{(1)} = & \frac{N_c C_F \alpha_s}{\pi^2 \rho^{1/2}} \frac{\Gamma_t}{m_t} \left\{ \left[Q_e^2 Q_t^2 + \frac{2Q_e Q_t v_e v_t}{1 - x_Z} + \frac{(a_e^2 + v_e^2) v_t^2}{(1 - x_Z)^2} \right] \right. \\
 & \times \left[\left(3 \ln \left(\frac{\sqrt{E^2 + \Gamma_t^2}}{\rho m_t} \right) + \frac{3}{2} + 6 \ln 2 \right) \frac{\pi^2}{\rho^{1/2}} + (18 + 24 \ln 2) \right] \\
 & + \frac{1}{s_w^4} \left[\frac{22}{3} + \frac{17\pi^2}{6} - \frac{17}{2} \ln 2 + (2 - 3\pi^2 + 9 \ln 2) \frac{3\sqrt{2}}{4} \ln(1 + \sqrt{2}) \right. \\
 & \left. \left. - \frac{27\sqrt{2}}{8} \left(\ln^2(1 + \sqrt{2}) + \text{Li}_2(2\sqrt{2} - 2) \right) \right] + \mathcal{O}(\rho) \right\}. \quad (3.2)
 \end{aligned}$$

Eq. (3.2) includes $1/\rho^{1/2}$ and $\ln(\rho/v^2)$ enhanced contributions. The $1/\rho^{1/2}$ power enhanced term is due to the Coulomb singularity in full analogy with the $1/v$ enhancement of the Coulomb gluon exchange in pNRQCD. An important difference with respect to pNRQCD is that since $\alpha_s/\rho^{1/2} \ll 1$ one does not need to resum the ρ -Coulomb corrections to all orders. The $\ln(\rho/v^2)$ term represents a new type of the large logarithms in the theory of top-quark threshold production. It originates from the logarithmic integral between the potential and ρ -potential momenta. Within the expansion by regions it shows up through the infrared divergence of the ρ -potential region and the ultraviolet divergence of the ordinary potential region. In the total result the divergences cancel each other leaving the logarithm of the scale ratio. Note that the contribution of the potential region alone is ultraviolet divergent. This is exactly the origin of the spurious divergences in pNRQCD description

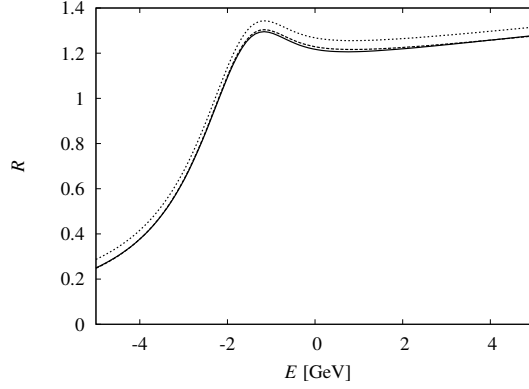


Figure 4: The normalized cross section of top-antitop production in electron-positron annihilation as function of the energy counted from the threshold. The dotted curve represents the leading order pNRQCD Coulomb approximation. The dashed curve includes the NLO nonresonant contribution and the solid curve includes the NNLO nonresonant correction as well. No strong coupling corrections to the resonance contribution are included.

of the unstable top production which does not include the ρ -potential region [30, 32]. Thus, the problem of the spurious divergences is naturally solved in ρ NRQCD.

3.2 Numerical results

The numerical effect of the nonresonant contribution on the total threshold cross section is shown in figure 4. In this plot we use the leading order pNRQCD approximation for the resonance contribution corresponding to the Coulomb Green’s function (2.4) with the strong coupling constant normalized at the soft scale $\mu_s = \alpha_s(\mu_s)C_F m_t$. The NLO contribution is negative and amounts to about 3.1% of the leading cross section above the threshold and competes with the LO contribution below the resonance region. The NNLO nonresonant contribution amounts to -0.9% at the threshold.

4. Summary

In this paper we presented the ρ NRQCD, a new nonrelativistic effective field theory which systematically accounts for the top-quark instability in the threshold top quark-antiquark pair production. The theory is based on the nonrelativistic expansion in the parameter $\rho = 1 - m_W/m_t$. It is optimized for high-order calculations and solves the problem of the spurious divergences of the standard pNRQCD approach. Within this framework we obtain the NLO nonresonant contribution to the total threshold cross section of the top quark-antiquark pair production in electron-positron annihilation and extended the analysis to the NNLO $\mathcal{O}(\alpha_s)$ nonresonant contribution which is computed to the leading order in ρ .

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